Unsteady effects in mechanically-excited turbulent plane jets

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Two methods of mechanically exciting a plane turbulent free jet are described; periodic perturbation of the nozzle exit velocity, and forced oscillation of a small vane located in the jet potential core. Hot-wire measurements obtained by conditional sampling techniques indicate that the flow fields of the two jets are substantially different although they have the same Strouhal number of 0.0032. While the mean flow development of the pulsed jet can be described adequately by a quasi-steady model, the vane-excited jet exhibits unsteady effects which depart significantly from quasi-steady approximations such as increased entrainment, amplification of excitation and non-linear effects in the form of the presence of high harmonics. The constancy of momentum flux has been examined in both the steady and unsteady jets

Keywords: *turbulent flow, plane jets, unsteady effects*

In the decade since the existence of large-scale coherent structures superimposed on a background of 'turbulence' in a plane mixing layer was demonstrated by Brown and Roshko[1], considerable effort has been directed at identifying similar structures in other flow configurations and determining their roles in establishing the basic characteristics of the flow field. Crow and Champagne[2] reported that in an axisymmetric jet, the most dominant and frequently occurring of all large-scale coherent structures, known as the 'preferred mode', corresponded to a Strouhal number, St, of about 0.3; these structures can be enhanced by introducing controlled acoustic excitation at $St = 0.3$. Results of such controlled excitation experiments[3-5] provide improved understanding of fundamental phenomena such as the structure of turbulence, shear-layer instability, and the natural entrainment process. On the other hand, excitations introduced at any arbitrary frequency may produce non-naturally occurring structures which may alter the characteristics of the flow field and may have potential practical applications in areas such as noise reduction, fluidics and combustion systems. This work arises from the need to enhance jet entrainment and mixing for developing compact, yet efficient, thrust augmenting ejectors for aerospace propulsion systems, particularly for VSTOL aircrafts.

Many jet excitation techniques have been reported, such as acoustic excitation[2,3,5,6], fluidic excitation[7,8] and mechanical excitation[9-12]. Two mechanically excited plane turbulent free jets are described here; a periodically pulsed jet and a vane-excited jet. The results for these two excited jets at a low Strouhal number of 0.0032 are compared and the unsteady effects are highlighted with reference to a quasi-steady jet model. The controversial question of momentum conservation in a jet is examined in both the steady and unsteady jet.

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Experimental set-up and measurement techniques

Periodically-pulsed jet

Measurements in both a steady and unsteady jet were made in the jet facility shown schematically in Fig 1. The nozzle, with a width (h) of 5 mm, has an aspect ratio of 60. Side plates were used to enhance the two-dimensionality of the jet so that at 100 nozzle widths downstream of the nozzle, the centre-line velocity variation over the central 50 mm of the length of the steady-jet was less than 2%.

In the unsteady jet, excitation was introduced in the stream-wise (x) direction by periodic perturbation of the nozzle exit velocity. This was achieved by venting air from the plenum chamber through an orifice, the area of which was varied sinusoidally by a sliding valve driven by an electromagnetic vibrator.

Instantaneous velocity measurements across the jet and up to 80 nozzle widths downstream were obtained with a constant temperature hot-wire anemometer. The single platinum alloy hot wire, 10 μm in diameter by 3 mm long, was operated at a resistance ratio of 1.3 and aligned parallel to the length of the nozzle. The linearized hot-wire anemometer output was recorded on FM tapes and subsequently analyzed on an EAI600 hybrid computing

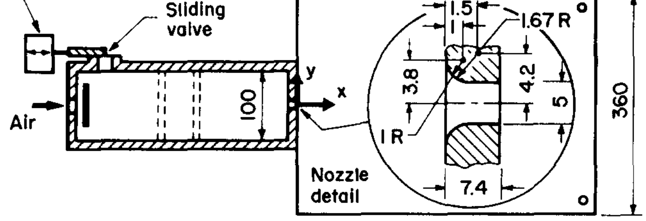

Fig 1 Pulsed-jet facility (all dimensions given in mm)

system using phase-averaging techniques as described by Lai and Simmons[11].

The mean jet exit Reynolds number was 1.1×10^4. Pulsation frequency (f) of 20 Hz with a zero-to-peak amplitude of pulsation of the nozzle exit velocity on the centreline of 6.9% of the mean nozzle exit velocity ($\bar{U}_e$) was used.

Vane-excited jet

In this jet facility (Fig 2), an excitation was introduced in the transverse (y) direction by using an electromagnetic vibrator to oscillate a vane in pitch about an axis 3 mm aft of its leading edge. The vane which had a symmetric airfoil section with a thickness of 1.3 mm, a span of 360 mm and a chord of 10 mm was located symmetrically in the potential core of the jet. The leading edge of the vane was 0.66 h from the nozzle. The mean jet exit Reynolds number was 1×10^4. Instantaneous velocity measurements were made with a DISA 55P51 X-wire probe and two DISA 55M constant temperature hot-wire anemometers for a vane frequency of oscillation (f) of 20 Hz and two vane amplitudes of oscillation (zero-peak) (ε) of 2.6° and 5.2° respectively. The details of the measurement techniques and data reduction scheme and the typical results for a range of other tested frequencies and amplitudes of oscillation have been reported by Lai and Simmons[13].

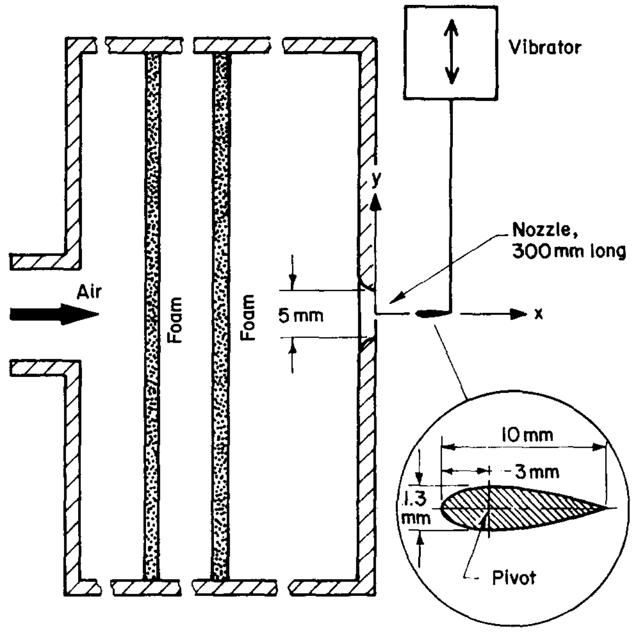

Fig 2 Vane-excited jet facility

Quasi-steady jet models

As both unsteady jets are operated at a low Strouhal number of 0.0032, it is appropriate to examine the results in the light of a quasi-steady jet model. By means of the usual triple decomposition, the instantaneous streamwise velocity U_i can be expressed in terms of a mean component $\bar{U}$, a periodic fluctuating component u due to the controlled excitation and the turbulent fluctuation u' as:

$$U_i = \bar{U} + u + u' \tag{1}$$

By defining:

$$\tilde{U}(x, y, t) = E(U_i) \tag{2}$$

where E is the ensemble expectation factor, then:

$$\tilde{U}(x, y, t) = \bar{U} + u \tag{3}$$

The distribution of $\tilde{U}$ over y for a given x and t is referred to here as an instantaneous velocity profile.

Periodically-pulsed jet

In this unsteady jet, the excitation produces a sinusoidal perturbation of the nozzle mass flow. Consequently, the instantaneous velocity profile $\tilde{U}(y, t)$ for a quasi-steady jet model may be expressed as:

$$\tilde{U}(y, t) = \bar{U}_s(y)(1 + \varepsilon \sin 2\pi f t) \tag{4}$$

where $\bar{U}_s(y)$ is the mean velocity profile of the steady jet, $\varepsilon = 0.069$, and $f = 20$ Hz. From Eq (4), it is obvious that $\bar{\tilde{U}}(y) = \bar{U}_s(y)$ and that all mean flow characteristics such as

Notation

F	Peak-to-peak velocity variation expressed as a percentage of mean velocity
f	Frequency of excitation, Hz
h	Nozzle width, mm
L	Streamwise length scale
l	Transverse length scale
P	Pressure
Q	$\int_{-\infty}^{\infty} \bar{U}\, dy$
Q_s	$\int_{-\infty}^{\infty} \bar{U}_s\, dy$
Q_E	Nozzle volume flow per unit nozzle length
Re	Reynolds number, $u'l/\nu$
St	Strouhal number, $fh/\bar{U}_e$
t	Time
T	Period of excitation
U_i	Instantaneous streamwise velocity
$\tilde{U}$	Phase-averaged streamwise velocity
$\bar{U}$	Mean streamwise velocity
u	Periodic streamwise fluctuation
u'	Streamwise turbulent fluctuation
x	Streamwise coordinate
y	Transverse coordinate
$y_{1/2}$	Jet half-width (distance from centre-line to the location where $\tilde{U}/\tilde{U}_c = 0.5$)
δ	$\varepsilon x/y_{1/2}$
ε	Zero-peak amplitude of excitation
η	$y/y_{1/2}$
ν	Kinematic viscosity of fluid
ρ	Density of fluid

Subscripts

c	At centre-line
e	At nozzle exit
s	For steady jet
sc	At centre-line of steady jet
1/2	At jet half-width

centreline velocity decay rate and jet spreading rate should agree with those of the steady jet provided $St = 0.0032$ is within the limits of applicability of a quasi-steady model.

Vane-excited jet

The effect of the oscillation of the vane is to produce a forced flapping motion of the jet in which the centreline of the mean velocity profile is sinusoidally displaced about its average position. For $x/h > 5$, the mean velocity profile of the steady jet can be represented in non-dimensional form as:

$$\frac{\bar{U}_s(\eta)}{U_{sc}} = \exp[-0.693\eta^2] \qquad (5)$$

where $\eta = y/y_{1/2}$ and $\bar{U}_{sc}$ is the centreline velocity of the steady jet.

By assuming the velocity profile in Eq (5) being displaced sinusoidally about a mean position in space, a quasi-steady model can be formulated by specifying the instantaneous velocity profile as:

$$\frac{\tilde{U}(\eta,t)}{\tilde{U}_c(t)} = \exp[-0.693(\eta - \delta \sin 2\pi f t)^2] \qquad (6)$$

where $\delta = \varepsilon x/y_{1/2}$ is the normalized transverse displacement of the centreline of the mean velocity profile from the average position, $\tilde{U}_c(t)$ is the instantaneous centreline velocity, and $f = 20$ Hz. Since the measured steady jet spreading rate $dy_{1/2}/dx$ is 0.113, it may be approximated that:

$$\delta = \begin{cases} 0.4 \\ 0.8 \end{cases} \quad \text{for} \quad \varepsilon = \begin{cases} 2.6° \\ 5.2° \end{cases} \qquad (7)$$

Eq (6) can be simulated using Eq (7) to give quantities such as the instantaneous variation of normalized velocity at $\eta = \eta_c$ given by $\tilde{U}(\eta_c, t)/\bar{U}(\eta_c)$ and the ratio of the rate of mean volume flow per unit width to that of the steady jet given by:

$$Q/Q_s = \int \bar{U}(\eta)dy / \int \bar{U}_s(\eta)dy.$$

Results

Mean quantities

In the periodically-pulsed jet, the measured instantaneous velocity profiles and mean velocity profiles, which are made non-dimensional using the centreline velocity ($\tilde{U}_c$) and jet half-width ($y_{1/2}$), all collapse onto the steady jet curve. The mean centreline velocity decay and the mean jet spreading with streamwise distance agree with those of the steady jet. Consequently, as shown in Fig 3, the mean entrainment, defined here as $(Q(x) - Q_E)/Q_E$, follows that of the steady jet. Here $Q(x)$ is the volume flow per unit width at a streamwise distance x obtained by integrating the corresponding mean velocity profile and Q_E is the nozzle volume flow per unit nozzle length.

Contrary to the periodically pulsed jet, the mean flow characteristics of the vane-excited jet at the same Strouhal number of 0.0032 cannot be predicted by the quasi-steady model (Eq (6)). As reported by Lai and Simmons[13], the mean centreline velocity decays faster and the mean jet spreading increases significantly compared with the corresponding steady jet values. The resulting entrainment is significantly higher than the corresponding steady jet value; for example, for $\varepsilon = 5.2°$ and $f = 20$ Hz, it is about 70% higher at $x/h = 20$ (Fig 3). It must be pointed out that a velocity profile, obtained by a velocity transducer which has a mean square response such as Pitot tube, is not the true mean velocity profile. As a result, entrainment values derived from such velocity profiles will be grossly in error for an unsteady jet. For example, the quasi-steady model indicates that pitot tube measurements overestimate entrainment value by 15% for $\delta = 0.8$ compared with measurements that yield the true mean velocity profile.

As shown in Fig 4(a), the quasi-steady model predicts the mean velocity profile follows that for the steady jet. However, the measured non-dimensional mean velocity profile is much 'flatter' near the centreline than that for the steady jet. Similar results were obtained by Harch *et al*[14] using a two colour laser Doppler anemometer for different operating conditions of the excited jet. Furthermore at $x/h = 40$, where maximum amplification of excitation occurs, the mean velocity profile exhibits a slight double peak for $\delta = 0.4$ (Fig 4(b)). However, the quasi-steady model indicates that a double peak exists only if $\delta \geq 1.6$. For comparison purposes, the mean velocity profile for $\delta = 1.6$ and $\delta = 2$ predicted by the quasi-steady model are plotted in Fig 4(b). Hence the extent of flapping under dynamic conditions is larger than that predicted from a static condition.

Instantaneous velocity variation at the centre-line and half-width of the mean velocity profile

For the periodically-pulsed jet, the variation of the instantaneous velocity at the centreline (clm) and at the half-width (hwm) of the mean velocity profile with time are shown respectively in Figs 5 and 6 for $x/h = 20$ and $x/h = 60$. It is obvious that the form of variation follows the trend predicted by the model but there are some slight

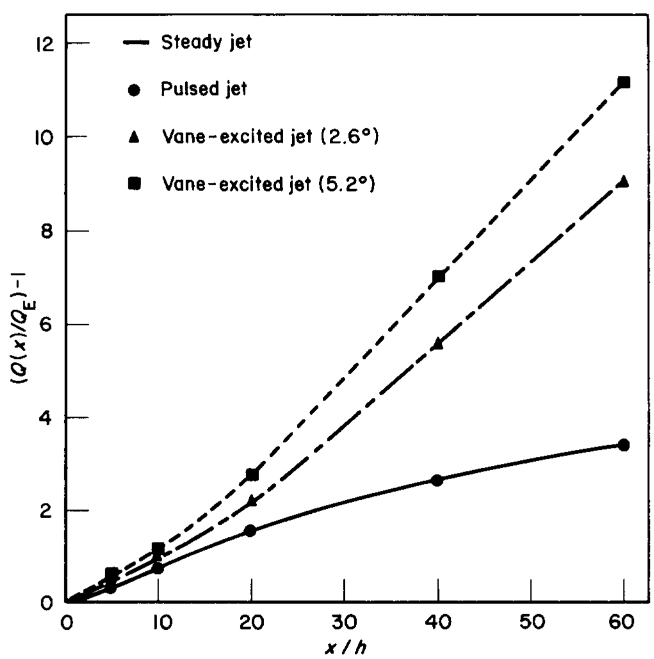

Fig 3 Variation of jet entrainment with streamwise distance

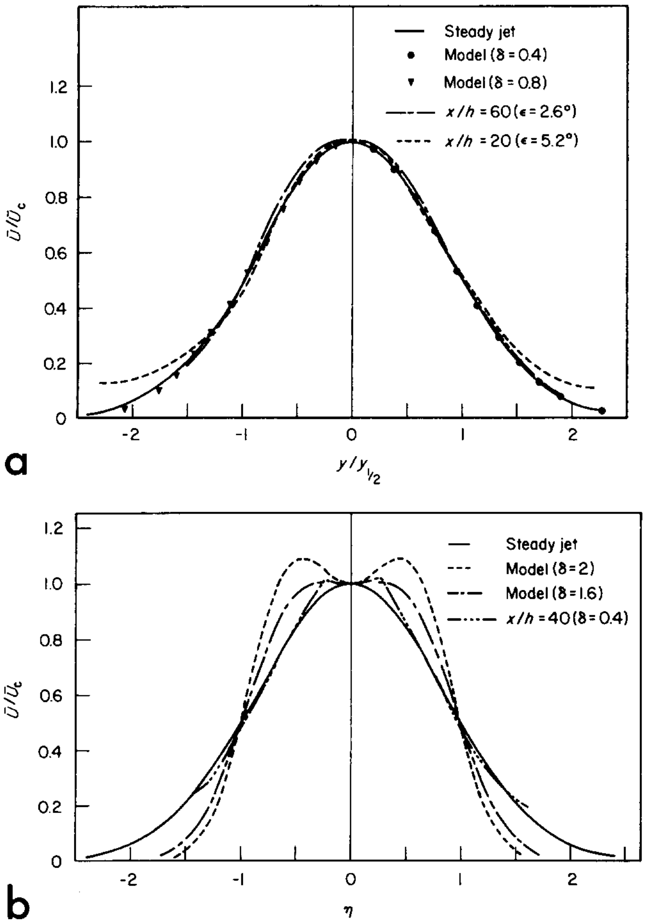

Fig 4 Non-dimensional velocity distributions

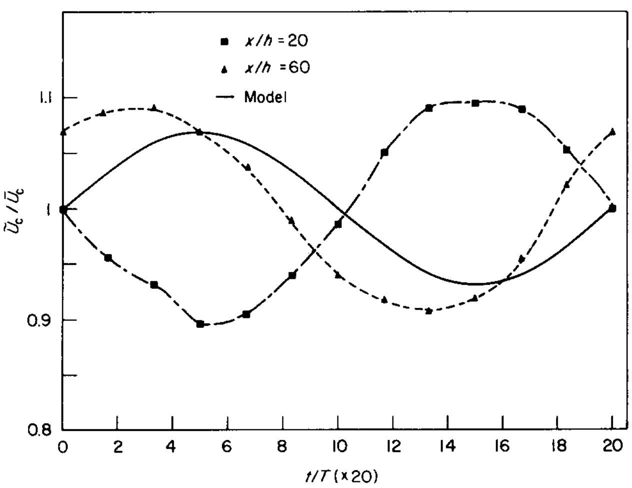

Fig 5 Variation of velocity at centreline of mean velocity profile (pulsed jet)

distortions in the waveform due to the presence of high harmonics. Furthermore there is some phase difference between the response at the centreline (clm) and that at the half-width (hwm); the quasi-steady model (Eq (4)) assumes no such phase difference. If F denotes the peak–peak amplitude variation of velocity expressed as a percentage of the mean velocity, then (Fig 7) the variation of F at the centre-line (F_c) with streamwise distance x/h is quite different from that at the half-width ($F_{1/2}$), further indicating the inadequacy of the model to predict instantaneous quantities. However, excitation in this case is not sufficiently amplified to cause any significant departure of the mean flow characteristics from the steady jet results.

In the vane-excited jet, because of the distinct flapping motion of the jet, the frequency of the periodic variation of the velocity at clm is predicted by the quasi-steady model to occur at twice the excitation frequency (Fig 8). Indeed, as shown in Fig 8, this is supported by the experimental results which, however, exhibit considerable

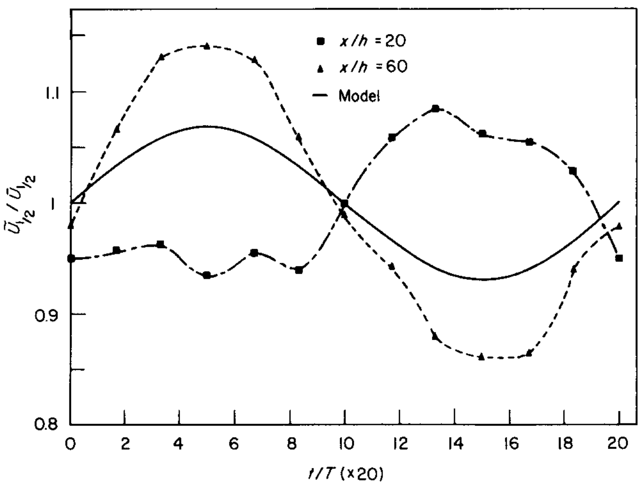

Fig 6 Variation of velocity at half-width of mean velocity profile (pulsed jet)

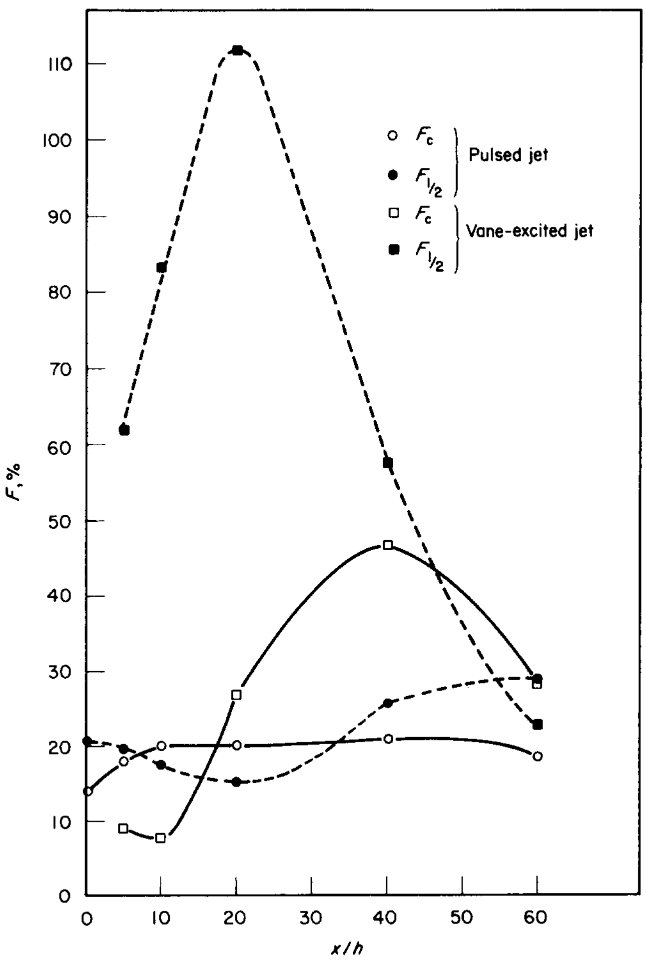

Fig 7 Variation of F *with streamwise distance*

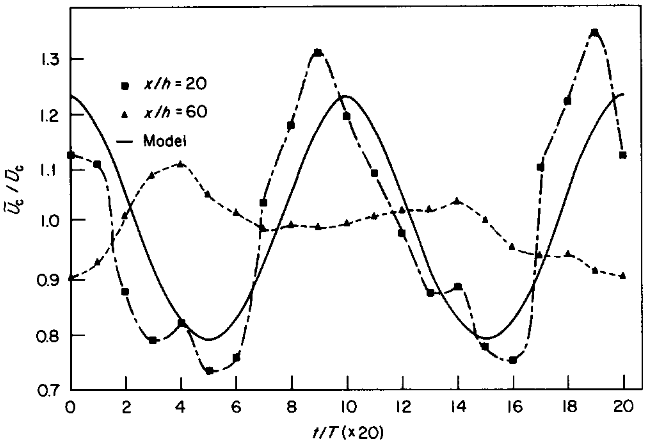

Fig 8 Variation of velocity at centreline of mean velocity profile (vane-excited jet)

distortions in the waveform from a pure sinusoidal form due to the presence of high harmonics. Fig 9 shows the variation of $\tilde{U}_{1/2}/\bar{U}_{1/2}$ with t/T for $x/h=20$ and $x/h=60$. The results confirm the prediction of the quasi-steady model that the frequency of the periodic variation of the velocity at hwm is the same as the excitation frequency. This phenomenon is typical of a distinct flapping motion. Again, there are significant distortions in the waveform. As observed in the periodically-pulsed jet, there is also a phase difference between the response of the jet at clm and hwm. The variation of F_c and $F_{1/2}$ with streamwise distance x/h in Fig 7 shows that the initial excitation is considerably amplified at some x/h beyond which it decays rapidly. For example, at $x/h=40$, F_c is $4\frac{1}{2}$ times its initial value! This large amplification of the excitation strongly affects the mean flow characteristics such as jet spreading and entrainment. In the unsteady jet of Simmons *et al*[10], the jet was deflected by oscillation of the nozzle, producing some form of flapping motion. They showed that for their tested Strouhal number range of 0.00001–0.00014, the unsteady jet behaviour conforms to that predicted by the quasi-steady jet behaviour. In the vane-excited jet reported here, although the Strouhal number is an order of magnitude higher than that of Simmons *et al*[10], it is still two orders of magnitude lower than the preferred mode of the corresponding steady jet and the resulting flow field is significantly different from the quasi-steady behaviour.

Momentum conservation

The question of momentum flux invariance has been a subject of debate mainly because of conflicting experimental evidence. It is, therefore, appropriate to examine closely the momentum flux invariant terms derived from the Navier-Stokes equations especially for the unsteady plane jets. By using the triple decomposition of instantaneous velocity (Eq (1)) and time-averaging, the cross stream and stream-wise momentum equations that describe the flow field of an unsteady turbulent plane jet are given respectively by Eqs (8) and (9):

$$\overline{U\frac{\partial V}{\partial x}}+\overline{V\frac{\partial V}{\partial y}}+\frac{\partial}{\partial x}(\overline{u'v'})+\frac{\partial}{\partial y}(\overline{v'^2}) = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial y}+\nu\left(\frac{\partial^2 \bar{V}}{\partial x^2}+\frac{\partial^2 \bar{V}}{\partial y^2}\right) \quad (8)$$

$$\overline{U\frac{\partial U}{\partial x}}+\overline{V\frac{\partial U}{\partial y}}+\frac{\partial}{\partial x}(\overline{u'^2})+\frac{\partial}{\partial y}(\overline{u'v'}) = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x}+\nu\left(\frac{\partial^2 \bar{U}}{\partial x^2}+\frac{\partial^2 \bar{U}}{\partial y^2}\right) \quad (9)$$

By following Tennekes and Lumley[15] using an order of magnitude analysis, Eq (8) can be approximated by:

$$-\frac{1}{\rho}\frac{\partial \bar{P}}{\partial y}=\frac{\partial}{\partial y}(\overline{v'^2}) \quad (10)$$

This approximation involves the following assumptions:

$$(u'/U_m)=O(l/L)^{1/2} \quad (11)$$

and:

$$\frac{U_m}{u'}\frac{1}{Re}\frac{(l)}{L}\to O \quad \text{as} \quad (l/L)\to 0 \quad (12)$$

where U_m is the maximum jet velocity, l is the cross stream length scale, L is the streamwise length scale, $Re=u'l/\nu$ is the Reynolds number which is sufficiently large for Eq (10) to be satisfied, and O() denotes 'of the order of'. Integrating Eq (10), and assuming no imposed external pressure gradient, leads to:

$$\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x}=-\frac{\partial}{\partial x}(\overline{v'^2}) \quad (13)$$

By substituting Eq (13) into Eq (9) and invoking the assumptions given by Eqs (11) and (12), Eq (9) is reduced by an order of magnitude analysis to:

$$\overline{U\frac{\partial U}{\partial x}}+\overline{V\frac{\partial U}{\partial y}}+\frac{\partial}{\partial x}(\overline{u'^2}-\overline{v'^2})+\frac{\partial}{\partial y}(\overline{u'v'})=0 \quad (14)$$

Integrating Eq (14) with respect to y yields:

$$\frac{d}{dx}\int_{-\infty}^{\infty}(\overline{U^2}+\overline{u'^2}-\overline{v'^2})dy=0 \quad (15)$$

Consequently:

$$\rho\int_{-\infty}^{\infty}(\bar{U}^2+\overline{\tilde{u}^2}+\overline{u'^2}-\overline{v'^2})dy=M_t \quad (16)$$

where M_t is a constant.

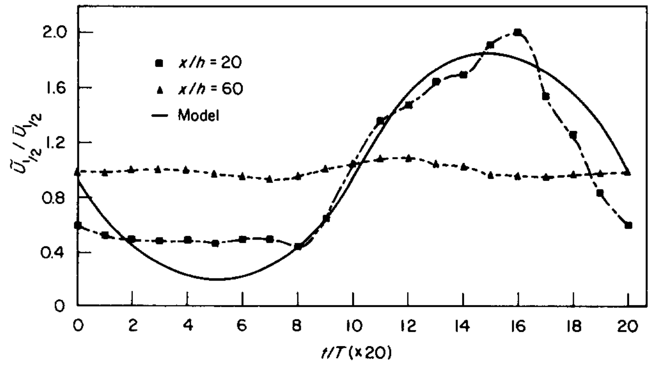

Fig 9 Variation of velocity at half-width of mean velocity profile (vane-excited jet)

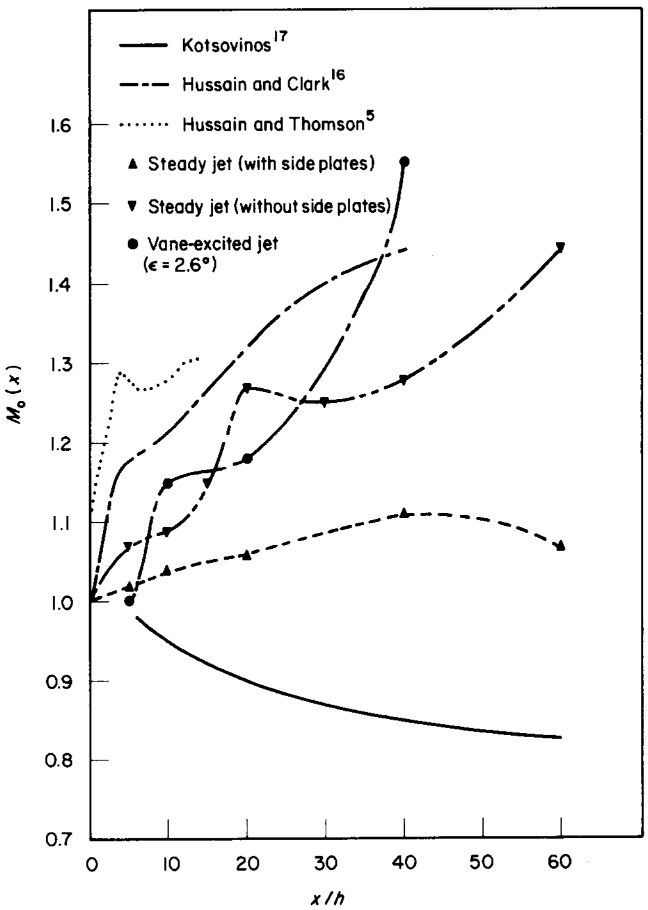

Fig 10 Variation of M_0 *with streamwise distance*

and:

$$\overline{\tilde{U}^2} = \bar{U}^2 + \overline{u^2}$$

Eq (16) is therefore a statement on the momentum flux invariant for an excited turbulent plane jet. It must be noted that the neglected terms in Eq (16) are of the order of (l/L) compared with the terms retained. Any departure from M_t being constant can be an indication of three-dimensional effects, reversed flow phenomenon, initial nozzle conditions and response of measuring equipment.

Steady jet

If there is no excitation introduced, $u=0$ so that Eq (16) reduces to:

$$\rho \int_{-\infty}^{\infty} (\bar{U}^2 + \overline{u'^2} - \overline{v'^2})\mathrm{d}y = M_t \tag{17}$$

It is generally assumed that $\overline{u'^2} \simeq \overline{v'^2}$ so that Eq (17) reduces to the following familiar momentum conservation for a steady jet:

$$\rho \int_{-\infty}^{\infty} \bar{U}\mathrm{d}y = M_0 \tag{18}$$

where M_0 is a constant.

Contrary to the expected constancy of M_0, experimental data reported for steady turbulent plane jets generally show that M_0 varies with streamwise distance. Fig 10 shows the variation of M_0 with streamwise distance from various workers. The data of Hussain and Clark[16] were taken from their N-50 case and the data of Kotsovinos[17] were the results of his model which fitted the data of some other workers. As shown in Fig 10, not only does M_0 vary with x/h, but the results from various workers show differing trends in that variation. It is necessary, therefore, to consider the factors which may affect the constancy of M_0.

Relative magnitude of $\overline{u'^2}$ *and* $\overline{v'^2}$

Table 1 lists out the contribution of $\overline{u'^2}$ and $\overline{v'^2}$ to the momentum flux of a steady turbulent plane jet for $x/h>40$. Here:

$$M_u = \rho \int \overline{u'^2}\mathrm{d}y \tag{19}$$

$$M_v = \rho \int \overline{v'^2}\mathrm{d}y \tag{20}$$

Although the absolute magnitude of M_u and M_v differs for data of different workers, the relative magnitude M_u/M_v is roughly 2. Hence, if the initial turbulence intensity is very low, ie $\overline{u_e'^2} \simeq \overline{v_e'^2} \simeq 0$, and if M_t (Eq (17)) is constant, then M_0 must decrease with x/h. The data of Kotsovinos follow this trend but the relative magnitude M_u/M_v can account for 10% of the momentum decrease at most.

Initial conditions

Hussain and Clark[16] reported that M_0 varied with different initial conditions at the nozzle and attributed the increase of M_0 with x/h to the presence of negative mean static pressure in turbulent jets. The data of Hussain and Thompson[5] also follow this trend. The increase of M_0 can be a result of flow reversal which cannot be distinguished from the main flow if measurements are made by hot-wire anemometers. In fact, the measurements of Goldschmidt *et al*[22] suggest that flow reversal occurs for $y/y_{1/2}$ greater than 1.6 and persists up to 60 nozzle widths downstream, with maximum flow reversal occurring at $x/h=20$.

Three-dimensional effects

The variation of M_0 with streamwise distance for a steady turbulent plane jet with and without side plates has been

Table 1 Relative contribution of $\overline{u'^2}$ and $\overline{v'^2}$ to momentum flux for steady turbulent plane jet

Source	Aspect ratio	M_u/M_o	M_v/M_o	M_u/M_v
This study	60	0.097	—	—
Chambers[18]	48	0.122	0.061	2
Everitt and Robins[19]	128	0.106	0.061	1.75
Gutmark and Wygnanski[20]	38.5	0.170	0.071	2.40
Heskestad[21]	132	0.143	0.077	1.86
Hussain and Clark[16]	44	0.110	—	—
Miller and Comings[22]	40	0.100	0.046	1.94

measured and plotted in Fig 10. The results indicate that while the jet with sides plates has a variation of M_0 of 10%, three-dimensional effects which are due to the absence of side plates, can cause a considerable increase in M_0.

For Eq (17) to be confirmed experimentally, all these factors must be accounted for and the velocity transducer and data reduction techniques must be capable of estimating $\bar{U}^2$, $\overline{u'^2}$ and $\overline{v'^2}$ separately.

Unsteady jet

Since the vane-excited jet measurements show significant increase in entrainment, it is important to check the experimental data against the constancy of momentum flux given by Eq (16). However, the difficulty of such an exercise has been outlined in the above section. When a jet is excited, effects such as flow reversal and three-dimensionality are expected to be more severe than the corresponding steady jet, rendering the problem even more difficult. Nevertheless, the data indicate that although M_u/M_0 ($\simeq 0.12$) and M_v/M_0 ($\simeq 0.067$) are slightly larger than the corresponding steady jet values, their relative magnitude (M_u/M_v) is about the same as in the steady jet. The variation of M_0 for $f = 20$ Hz and $\varepsilon = 2.6°$ with x/h is shown in Fig 10. Since the vane-excited jet does not have any side plates to enhance two-dimensionality, the increase in M_0 with x/h is quite acceptable compared with the corresponding steady jet. Furthermore, unlike the steady jet in which $l/L \simeq 0.1$ for $x/h > 20$, the term l/L varies considerably with x/h and does not reach a constant value until at very large distance downstream of the nozzle. In the vane-excited jet, l/L can be of the order of 0.2. Consequently, the terms left out in the momentum flux invariant in Eq (16) can amount to 20%.

Conclusions

Although the Strouhal numbers for the periodically pulsed and vane-excited jets are the same, the flow fields of the two jets are remarkably different. In the periodically pulsed jet, the mean quantities are adequately predicted by a quasi-steady jet model. This is expected for a jet of such low Strouhal number (0.0032). For the vane-excited jet, however, not only are unsteady effects observed in the instantaneous quantities such as F_c, but also the mean flow characteristics such as entrainment depart considerably from quasi-steady expectations. The results presented here indicate that an excitation introduced in the transverse direction of a jet is a much more effective means of enhancing entrainment than a streamwise excitation.

The momentum flux invariant term has been derived for an unsteady turbulent plane jet. The apparent lack of constancy of momentum exhibited in the data of various workers has been examined and is attributed to the combined effects of flow reversal, three-dimensionality and the difficulty of measuring the various terms in the momentum invariant accurately. These effects are more severe and more difficult to measure in the unsteady jet. Measurements are, therefore, required to account for these effects.

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